GRAVITY COUPLINGS IN THE STANDARD-MODEL EXTENSION

QUENTIN G. BAILEY

Physics Department, Embry-Riddle Aeronautical University
3700 Willow Creek Road, Prescott, AZ 86301, USA
Email: baileyq@erau.edu

The Standard-Model Extension (SME) is an action-based expansion describing general Lorentz violation for known matter and fields, including gravity. In this talk, I will discuss the Lorentz-violating gravity couplings in the SME. Toy models that match the SME expansion, including vector and two-tensor models, are reviewed. Finally I discuss the status of experiments and observations probing gravity coefficients for Lorentz violation.

1. Introduction

General Relativity (GR) and the Standard Model of particle physics provide a comprehensive and successful description of nature. Nonetheless, it is expected that an underlying unified description containing both theories as limiting cases exists, presumably at the Planck scale. So far, such a complete unified theory remains largely unknown. Moreover, direct measurements at the Planck scale are infeasible at present so experimental clues about this underlying theory are sparse.

One promising approach is to study suppressed effects that may come from the underlying theory. An intriguing class of signals that are potentially detectable in modern sensitive experiments are minuscule violations of local Lorentz symmetry.[1] A comprehensive effective field theory framework exists called the Standard-Model Extension (SME)[2,3] that describes the observable signals of Lorentz violation. In this framework, the degree of Lorentz violation for each type of matter or field is controlled by its coefficients for Lorentz violation, which vanish when Lorentz symmetry holds.

So far, theoretical and experimental work on the SME has mostly involved the Minkowski-spacetime limit.[4] Lorentz violation in the gravitational sector remains comparatively unexplored. In this talk, we focus on

two basic types of Lorentz violation involving gravity: pure-gravity couplings and matter-gravity couplings. For a more detailed discussion of these topics, the reader is referred to Refs. 5–7.

2. Theory

The SME with both gravitational and nongravitational couplings was presented in the context of a Riemann-Cartan spacetime in Ref. 3. In the matter sector of the SME, Dirac spinor fields can be used for describing the matter-gravity couplings that are expected to dominate in many experimental scenarios. In this limit the Lagrange density takes the form

$$\mathcal{L}_m = \tfrac{1}{2} i e e^{\mu}{}_{a} \overline{\psi} (\gamma^a - c_{\nu\lambda} e^{\lambda a} e^{\nu}{}_{b} \gamma^b + ...) \overleftrightarrow{D}_{\mu} \psi - e \overline{\psi} (m + a_{\mu} e^{\mu}{}_{a} \gamma^a + ...) \psi + ..., \quad (1)$$

where the ellipses represent additional coefficients in the SME omitted here for simplicity. The standard vierbein ($e_{\mu}{}^{a}$) formalism is used to incorporate the spinor fields ψ and the gamma matrices γ^a into the tangent space at each point in the spacetime. Both the spacetime connection and the $U(1)$ connection are included in the covariant derivative. The quantities $c_{\mu\nu}$ and a_{μ} are species-dependent coefficients for Lorentz violation.

In the Riemann-spacetime limit, the Lagrange density for the pure-gravity sector of the SME takes the form

$$\mathcal{L}_g = \frac{1}{2\kappa} e [(1-u)R + s^{\mu\nu} R^T_{\mu\nu} + t^{\kappa\lambda\mu\nu} C_{\kappa\lambda\mu\nu}] + \mathcal{L}'. \quad (2)$$

The 20 coefficients for Lorentz violation u, $s^{\mu\nu}$, and $t^{\kappa\lambda\mu\nu}$ control the leading Lorentz-violating gravitational couplings in this expression. The curvature tensors appearing are the Ricci scalar R, the trace-free Ricci tensor $R^T_{\mu\nu}$ and the Weyl conformal tensor $C_{\kappa\lambda\mu\nu}$. By convention $\kappa = 8\pi G$, where G is Newton's gravitational constant. The additional term $\mathcal{L}'$ contains the matter sector and possible dynamical terms governing the 20 coefficients. General coordinate invariance is maintained by the SME action while local Lorentz transformations and diffeomorphisms of the matter and gravitational fields are not respected by the SME action when $\mathcal{L}' = 0$.

Some geometric constraints arise when Lorentz violation is introduced in the context of Riemann-Cartan geometry. When the coefficients for Lorentz violation in the matter and gravity sectors are nondynamical or prescribed functions this generally conflicts with the Bianchi identities. However, when the coefficients arise through a dynamical process, conflicts with the geometry are avoided.[3] This includes spontaneous Lorentz-symmetry breaking scenarios. The coefficients for Lorentz violation are treated as arising from spontaneous Lorentz-symmetry breaking in the approach of Refs. 5–7.

It is generally a challenging task to study the gravitational effects in Eqs. (1) and (2) in a generic, model-independent way. It turns out that some simplifications to the analysis arise in the linearized gravity regime and it is then possible to extract effective linearized Einstein equations and modified equations of motion for matter, under certain assumptions on the dynamics of the coefficients for Lorentz violation. These equations then only involve the vacuum expectation values of the coefficients for Lorentz violation which are denoted as $\overline{a}_\mu$, $\overline{c}_{\mu\nu}$, and $\overline{s}_{\mu\nu}$. Due to species dependence, $\overline{a}_\mu$ and $\overline{c}_{\mu\nu}$ contain 12 and 27 independent coefficients for ordinary matter, respectively. In the pure-gravity sector, only the 9 species-independent $\overline{s}_{\mu\nu}$ coefficients appear in the linearized gravity limit.

In the post-newtonian limit, the metric for the SME can be constructed from the effective Einstein equations. An interesting feature arises that terms in the metric acquire a novel species dependence from the $\overline{a}_\mu$ and $\overline{c}_{\mu\nu}$ coefficients. One can also attempt to match to the standard Parametrized Post-Newtonian (PPN) formalism.[8] This involves constraining $\overline{s}_{\mu\nu}$ to an isotropic form in a special coordinate system with only one independent coefficient $\overline{s}_{00}$. Therefore there is a partially overlapping relationship between the two approaches, and the SME offers new types of signals for gravitational tests.[5]

3. Toy models

Several models of spontaneous Lorentz-symmetry breaking exist that have a connection to the general formalism described above. The simplest types of models involve a dynamical vector field B_μ that acquires a vacuum expectation value b_μ via a potential term in the lagrangian, which are generically called bumblebee models. Bumblebee models can produce effective $s_{\mu\nu}$, $c_{\mu\nu}$, and a_μ terms.[5,7] Another interesting class of models involves an antisymmetric two-tensor field $B_{\mu\nu}$.[9] The modes appearing in a minimal version of these models can include a scalar as well as nondynamical massive modes, in addition to producing a background vacuum expectation value $b_{\mu\nu}$. Furthermore, flat spacetime theories with a self interacting $B_{\mu\nu}$ field may only be stable and renormalizable when the potential admits a nontrivial minima $b_{\mu\nu}$, thus spontaneously breaking Lorentz symmetry. When nonminimal couplings to gravity are included, these models can also produce effective $\overline{s}_{\mu\nu}$ coefficients. Furthermore, it can be shown that these effective $\overline{s}_{\mu\nu}$ coefficients cannot be reduced to an isotropic form, and so lie outside of PPN analysis.

4. Matter-gravity tests

The dominant effects from the coefficients $\overline{a}_\mu$ and $\overline{c}_{\mu\nu}$ are modified equations of motion for bodies interacting gravitationally. Due to the particle species dependence of these coefficients, the motion of a macroscopic body in a gravitational field will depend on its internal composition. This constitutes a violation of the weak equivalence principle (WEP), so the coefficients control WEP violation as well.[7] Existing and proposed tests that can probe these coefficients include ground-based gravimeter, atom interferometry, and WEP experiments. Also of interest are lunar and satellite laser ranging observations as well as measurements of the perihelion precession of the planets.

Among the most sensitive tests are proposed satellite missions designed to test WEP in a microgravity environment. The observable of interest for these tests is the relative acceleration of two test bodies of different composition. When the relative acceleration is calculated in the satellite reference frame in the presence of SME coefficients $\overline{a}_\mu$ and $\overline{c}_{\mu\nu}$, some interesting time-dependent effects arise. The standard reference frame for reporting coefficient measurements in the SME is the Sun-centered celestial equatorial reference frame or SCF for short.[10] Upon relating the satellite frame coefficients to the SCF, oscillations in the relative acceleration occur at a number of different frequencies including multiples and combinations of the satellite's orbital and rotational frequencies, as well as the Earth's orbital frequency. This time dependence allows for the extraction of Lorentz-violating amplitudes independently of the standard tidal effects. Future space-based WEP tests offer sensitivities ranging from 10^{-7} GeV to 10^{-16} GeV for $\overline{a}_\mu$ and 10^{-9} to 10^{-16} for $\overline{c}_{\mu\nu}$. Of particular interest are the STEP,[11] MicroSCOPE,[12] and Galileo Galilei[13] experiments.

5. Pure-gravity sector tests

The primary effects due to the nine coefficients $\overline{s}_{\mu\nu}$ in the pure-gravity sector of the SME can be obtained from the post-newtonian metric and the standard geodesic equation for test bodies.[5] Tests potentially probing these coefficients include Earth-laboratory tests with gravimeters, torsion pendula, and short-range gravity experiments. Space-based tests include lunar and satellite laser ranging, studies of the secular precession of orbital elements in the solar system and with binary pulsars, and orbiting gyroscope experiments.

Some analysis placing constraints on the $\overline{s}_{\mu\nu}$ coefficients has already

been reported. Using lunar laser ranging data spanning over three decades, Battat, Chandler, and Stubbs placed constraints on 6 combinations of the $\overline{s}_{\mu\nu}$ coefficients at levels of 10^{-7} to 10^{-10}.[14] The modified local acceleration on the Earth's surface was measured by Müller *et al.* using an atom interferometric gravimeter, resulting in 7 constraints on the $\overline{s}_{\mu\nu}$ coefficients at the level of 10^{-6} to 10^{-9}.[15]

Recently, the modifications of the classic GR time-delay effect due to the $\overline{s}_{\mu\nu}$ coefficients were studied.[16] By studying light propagation with the post-newtonian metric modified by the $\overline{s}_{\mu\nu}$ coefficients, the correction to the light travel time for a signal passing near a mass M has been obtained. Time-delay tests could be particularly useful for constraining the isotropic $\overline{s}_{TT}$ coefficient, and future tests could yield competitive sensitivities to the $\overline{s}_{JK}$ coefficients. Measurements of $\overline{s}_{\mu\nu}$ coefficients could be obtained by using data from time-delay tests such as Cassini and BepiColombo.[17] Also under study are modifications from $\overline{s}_{\mu\nu}$ coefficients to the classic light-bending formula in GR.[18]

References

1. R. Bluhm, Lect. Notes Phys. **702**, 191 (2006).
2. D. Colladay and V.A. Kostelecký, Phys. Rev. D **55**, 6760 (1997); Phys. Rev. D **58**, 116002 (1998).
3. V.A. Kostelecký, Phys. Rev. D **69**, 105009 (2004).
4. *Data Tables for Lorentz and CPT Violation,* V.A. Kostelecký and N. Russell, 2010 edition, arXiv:0801.0287v3.
5. Q.G. Bailey and V.A. Kostelecký, Phys. Rev. D **74**, 045001 (2006).
6. V.A. Kostelecký and J.D. Tasson, Phys. Rev. Lett. **102**, 010402 (2009).
7. V.A. Kostelecký and J.D. Tasson, arXiv:1006.4106v1.
8. C.M. Will, Living Rev. Relativity **9**, 3 (2006).
9. B. Altschul *et al.*, Phys. Rev. D **81**, 065028 (2010).
10. V.A. Kostelecký and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
11. T.J. Sumner *et al.*, Adv. Space Res. **39**, 254 (2007); P. Worden, these proceedings.
12. P. Touboul *et al.*, Comptes Rendus de l'Académie des Sciences, Series IV, **4**, 1271 (2001).
13. A.M. Nobili *et al.*, Exp. Astron. **23**, 689 (2009).
14. J.B.R. Battat *et al.*, Phys. Rev. Lett. **99**, 241103 (2007).
15. H. Müller *et al.*, Phys. Rev. Lett. **100**, 031101 (2008); K.-Y. Chung *et al.*, Phys. Rev. D **80**, 016002 (2009).
16. Q.G. Bailey, Phys. Rev. D **80**, 044004 (2009).
17. B. Bertotti, L. Iess, and P. Tortura, Nature **425**, 374 (2003); L. Iess and S. Asmar, Int. J. Mod. Phys. D **16**, 2191 (2007).
18. R. Tso, these proceedings.